THREE-NO REMAINDER-GUARANTEE!

<u>Math skills</u>: Division, addition

<u>You will need</u>: Pen and paper

<u>Difficulty level</u>: 2

<u>Introduction</u>: Tell your students that you can predict if a number divided by 3 will have a remainder or not, without a calculator!

<u>What to do</u>: Ask a volunteer to write a phone number or a zip code on the blackboard. Before he/she finishes writing the number you are going to tell if the number will divide by 3 with or without a remainder. If the number has a remainder, you can offer him/her another number that will divide by the number 3 with no remainder.

<u>How it works</u>: If the volunteer wrote: 21657, adding the digits gives 2+1+6+5+7=21 which is divisible by 3 with no remainder as is 21657. If you can divide the results by three with no remainder, the answer is that the original number will also be divisible by three with no remainder! It is even more exciting - Try to mix the digits and you will see that all the combinations will divide by 3 with no remainder as well! (Try 12567; 56721;76215; 67152; etc.) If your volunteer chooses a number that you see will have a remainder when divided by 3, just offer to add a new integer that will cause the new number to divide exactly by 3. For example: If a volunteer wrote 65123, the sum of the integers is 6+5+1+2+3=17 which is one short of 18 that is divisible by 3. If you offer to add one to the number to obtain 651231, this new number is now divisible by 3 with no remainder. Note that all of the examples are five digits numbers but the process will work for any whole numbers.

<u>Why it works</u>: Think of all the numbers that are divisible by 3. They could be written as a series: 3, 6, 9, 12, 15, etc. Note that in each number of the series the digits add up to a multiple of 3. When these sums form multi-digit numbers, their successive sums eventually are 3, 6, or 9. The reason for this is:

> the first 3 numbers of the series 3, 6, 9 are divisible by 3
> the next number in the series, 12, is obtained by adding 3 to 9
> whenever 9 is added to a number, the sum of the digits do not change because adding 9 adds a ten and removes a unit.

Therefore the sum of the integers in the series will become 3, 6, 9, and 3 etc, always divisible by 3.